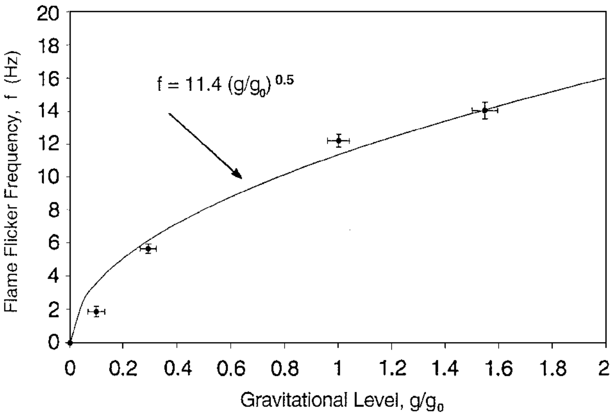

Fig. 2 Flame flicker frequency as a function of relative gravitational level.

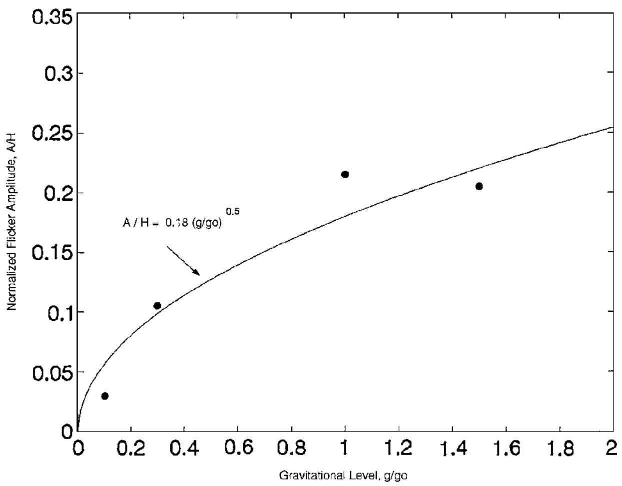

Fig. 3 Nondimensional flicker amplitude (with respect to the corresponding measured average flame height) as a function of gravitational level.

burning in air) supports the observation in the literature (e.g., Ref. 2) that flicker frequency may not be sensitive to fuel type.

Figure 3 shows the amplitude A of the flame height oscillation (nondimensionalized with respect to the corresponding measured average flame height H) as a function of gravity. The amplitude is found to scale approximately as

$$A/H = 0.18(g/g_0)^{0.5} \tag{3}$$

which shows a $g^{0.5}$ dependence for flicker amplitude as well.

Conclusions

An experimental study of diffusion flame flicker under different gravitational levels has provided a correlation between the flicker frequency and gravitational level in the form $f = 11.4(g/g_0)^{0.5}$, which agrees with one obtained from previous theoretical results.[6,7] It is shown that flicker is buoyancy-dominated and that flicker frequency increases with increasing gravitational level. In addition, it is shown that the nondimensional flicker amplitude increases with increasing gravitational level and has a $(g)^{0.5}$ dependence.

Acknowledgments

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J. P. Gore
Associate Editor

Static and Modal Analysis of Twin-Cell Box Girder Structures

Saeid A. Alghamdi*
King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

Nomenclature

D	=	plate flexure rigidity $Et^3/[12(1-\nu^2)]$
E	=	modulus of elasticity
M_w	=	warping moment
$\boldsymbol{S}'_n$	=	vector of stress resultants in local coordinate system
$\boldsymbol{T}^T$	=	transpose of transformation matrix
t	=	plate thickness
$u_{,y}$	=	derivative with respect to y coordinate
$w^{(4)}$	=	fourth derivative of transverse displacement with respect to longitudinal coordinate x
α	=	cross-section aspect ratio
$\bar{\alpha}$	=	square root of the ratio of primary to secondary rigidities
β	=	ratio of web thickness to flange thickness
$\boldsymbol{\Delta}'_n$	=	vector of nodal displacements in local coordinate system
ν	=	Poisson's ratio

Introduction

FOR reasons of economy, efficiency, and practicality, the use of thin-walled box-type structures is considered an ingenious

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*Assistant Professor of Structural Engineering; currently Visiting Fulbright Researcher, Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061.

utilization of structural material. In different forms these cellular structures (Figs. 1a and 1b) have been used extensively to build civil and mechanical structures such as highway bridges, railway bridges, and aerospace structures. It is noted that thin-walled structures[1−4] manifest peculiar patterns of stresses and deformations including 1) nonuniform warping stresses caused by nonuniform torsion[5,6]; 2) shear lag resulting from Web deformations[4,6]; and 3) transverse deformations resulting from eccentric loads and magnification of normal and shear stresses at junctures of webs and flanges, and the Saint-Venant's theory of beam bending cannot properly describe the static behavior of these structures.[3,7,8] These unique characteristics have been studied to a reasonable extent with various assumptions and approximations using 1) method of thin-walled beams[6−10]; 2) method of folded plates[1,3,11]; 3) method of finite strips[6,12−14]; and 3) method of finite elements.[2,5,15−18] The state of literature available on the mechanics and design of these structures does however indicate a need for more conclusive and refined studies to build a design database to establish a set of guidelines that can be used to optimize designs of these structures.

This Note presents 1) highlights of key previous studies on cellular structures; 2) selected results of design-oriented finite element method (FEM) studies on the static and dynamic characteristics of thin-walled twin-cell box-girders; and 3) a set of recommendations for proportioning of these structures. The results and recommendations presented are expected to form a basis for development of design guidelines to proportion more optimal box-girder structures.

Governing Differential Equations

Within the framework of linear elasticity, a box-girder (Fig. 1a) is modeled as a composition of plates using the theory of plane stress and plate bending to study the characteristics of its structural response. A typical plate element (shown in Fig. 1b) is used to express the relationships of elemental stresses, stress resultants to edge displacement. The resulting biharmonic differential equations for plate bending and membrane actions are solved for the deflection function $w(x, y)$ and the Airy stress function $\varphi(x, y)$ using a Fourier series. The deflection function for a plate simply supported on two opposite edges and free on the other two edges (for example) is written as

$$w(x, y) = \sum_{n=1}^{\infty} a_n(y) \sin \frac{n\pi x}{l} \tag{1}$$

in which $a_n(y)$ terms of the series are determined for specified boundary conditions. The repeated applications of unit deformations along the free edges lead to a system of algrebraic equations to solve for $a_n(y)$. The stress resultants for bending, torsion, and shear effects are determined from the following expressions:

$$M_{an} = -D(w_{,aa} + \nu\, w_{,ll}) \tag{2a}$$

$$M_{aln} = -D(1 - \nu)w_{,al} \tag{2b}$$

$$Q_{ln} = -D(w_{,aaa} + w_{,all}) \tag{2c}$$

The generic subscripts a and l correspond to both x coordinate and y coordinate to represent bending moments M_{xn} and M_{yn} and shear forces Q_{xn} and Q_{yn}. Generalized Hooke's law and the strain-displacement relationships

$$\varepsilon_{xn} = (1/E)(\sigma_x - \nu\sigma_y) \tag{3a}$$

$$\gamma_{xyn} = u_{,y} + v_{,x} \tag{3b}$$

are then used to determine membrane displacements u_n and v_n using the following integral expressions:

$$u_n = \int \varepsilon_{xn}\, \mathrm{d}x \tag{4a}$$

$$v_n = \int (\gamma_{xyn} - u_{,y})\, \mathrm{d}y \tag{4b}$$

Equations (4) are then used to formulate the force-displacement relationships for local membrane actions in a plate element. The analysis and interpretation of results require the use of one global coordinate system to build the following force displacement and global to local force matrix relationships:

$$\boldsymbol{S}_n = \boldsymbol{T}^T \boldsymbol{k}_n \boldsymbol{T}^T \boldsymbol{\Delta}_n \tag{5a}$$

$$\boldsymbol{S}'_n = \boldsymbol{T}^T \boldsymbol{S}_n \tag{5b}$$

for the nth term of the Fourier series of response quantities.

Numerical Studies

In studies of single-box structures, it has been noted by several researchers[14,18,19] that 1) shear lag is a design consideration that should be accounted for carefully in the analysis of thin-walled cellular structures; 2) stiffeners of flanges and webs play a major role on stress distributions in these structures and the amount of stiffeners area should be selected carefully; and 3) rigidity of end diaphragms and/or internal diaphragms play a fundamental role in modifying characteristics of structural distortions, stress distributions, and the characteristics of modal vibrations. A full account of these key

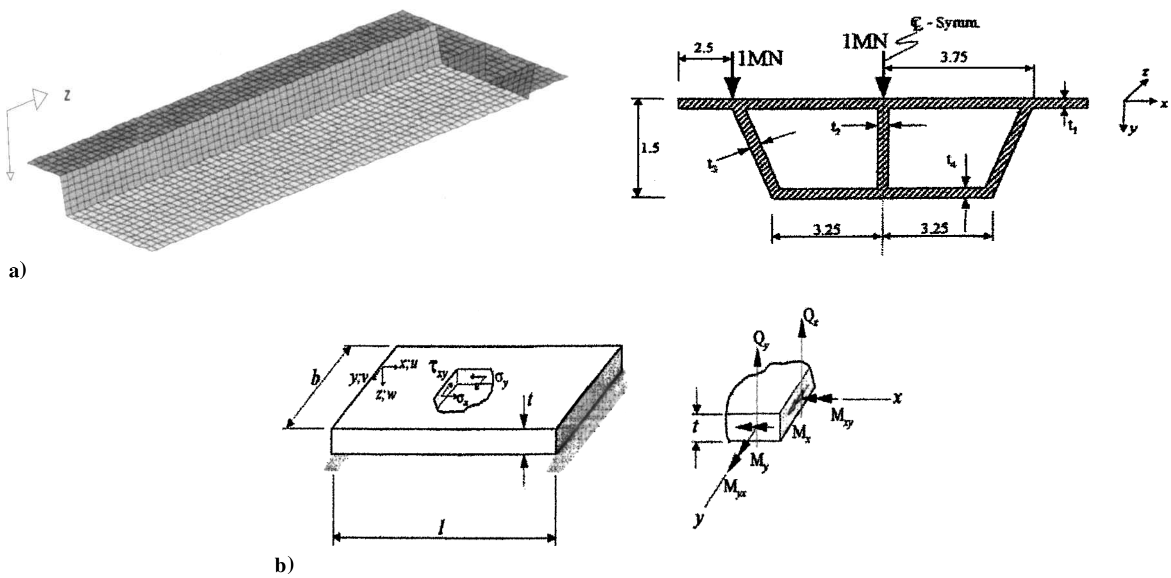


Fig. 1 Twin-cell box girder structure: a) FEM model and the cross section and b) components of displacements, stress resultants, and stresses in a plate element.

design issues requires results from more extensive FEM studies, but the available literature includes only a few studies that have been limited mostly to studying the general aspects of shear lag phenomenon. The following sections present results of the first stage of a study initiated by the author[20] to explore the influence of key design issues to investigate 1) shear lag phenomenon and its influence on stress distributions; 2) characteristics of modes of vibration; 3) effects of torsional rigidity of cross section; and 4) effects of rigidity of cross-section diaphragms on modal characteristics.

Torsion-Flexure Properties of Box Girders

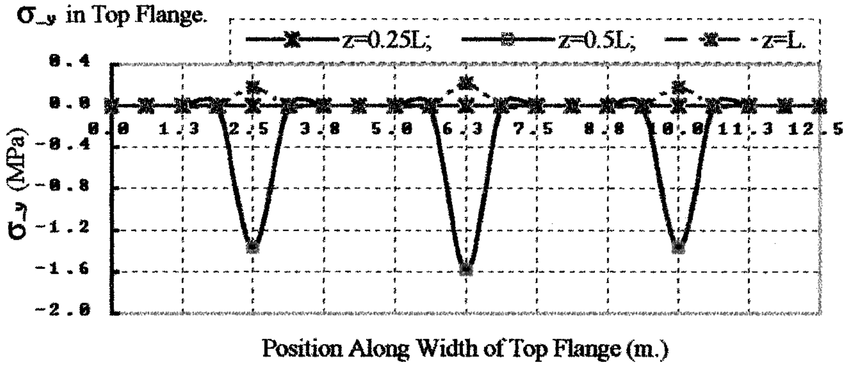
For a single cell with width b, depth $d = \alpha b$, flange thickness t_f, web thickness $t_w = \beta t_f$, the flexure and torsion rigidities of a twin-cell box-girder cross section is the superposition of contributions of the two cells. The ratio of torsion constant J to flexural constant I (i.e., $R = J/I$) is simplified to the following form:

$$R(\alpha, \beta) = 12\beta/(3+\beta^2)\alpha + (3+\alpha^2)\beta \tag{6}$$

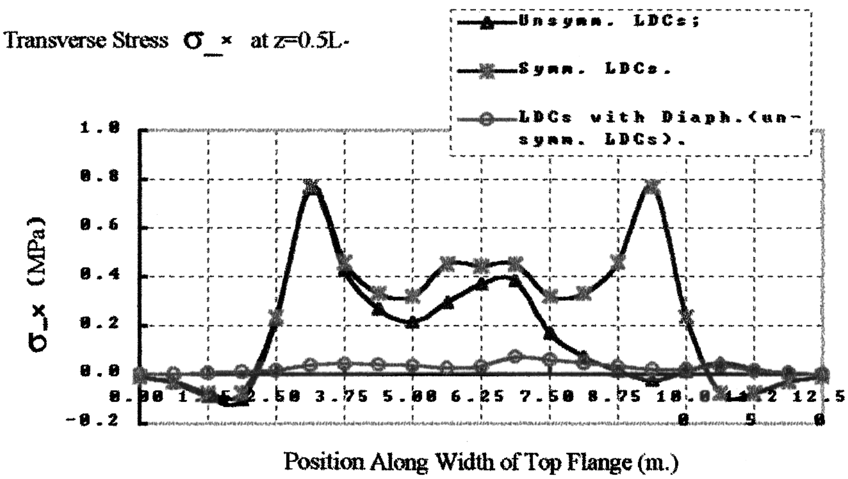
where this ratio 1) attains maximum value $R_{\max}$ at $\beta^* = \sqrt{(3)}$ regardless of the value of α and 2) approaches a maximum value of 4 at β^* as α approaches zero (flat cross section).

FEM Studies on the Structural Static and Modal Characteristics

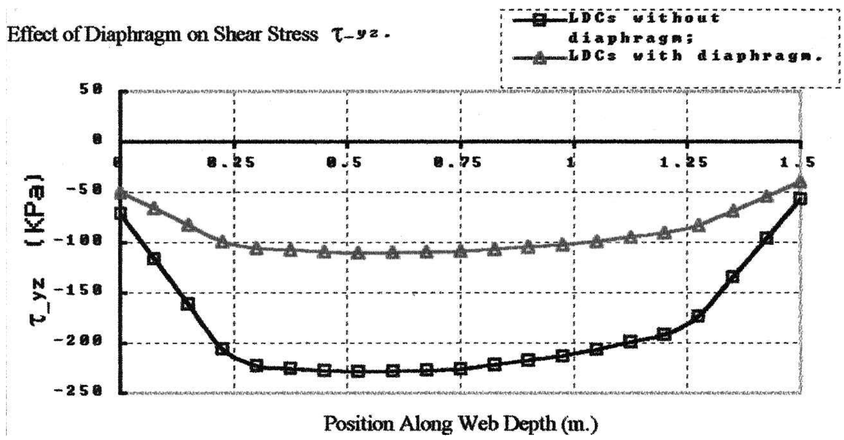
The FEM modeling and analysis of a two spans' continuous structure with the cross section shown in Fig. 1a are performed using a general-purpose computer code,[21] and the structural mesh is designed to facilitate the testing of suitability of four types of finite elements for the investigation of several combinations of load cases with/without cross-section diaphragms. The material and sectional properties used are modulus of elasticity $E = 34.5$ GPa; Poisson's ratio $\nu = 0.15$; mass density $\gamma = 2400$ kg/m^3; and plate thicknesses $t_1 = 0.2$ m, $t_2 = 0.420$ m, $t_3 = 0.425$ m, and $t_4 = 0.15$ m. Full details of studies conducted to investigate key characteristics of twin-cell



a) σ_y in top flange for symmetric and unsymmetric load cases with a diaphragm



b) σ_x in top flange for symmetric/unsymmetric loads



c) τ_{yz} in the middle web

Fig. 2 Shear lag effects for load cases with/without a diaphragm.

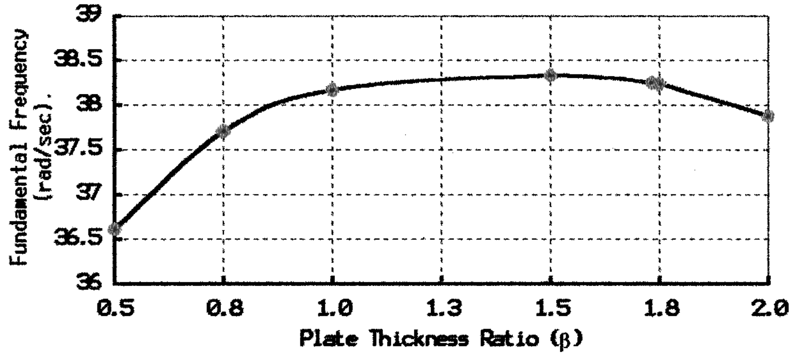
box girder structures are reported elsewhere,[20,22,23] and only a sample of results are presented in the following sections.

Several case studies of static and modal analysis were performed to investigate the influence of key design variables (including relative strength of constituent plate elements using plate thickness ratio β; use of cross-section diaphragms; type of finite elements used to model structure; modeling of supports conditions; and inclination of cross-section webs) on stress distributions and modal characteristics. A sample of the results obtained is summarized in Figs. 2 and 3, respectively.

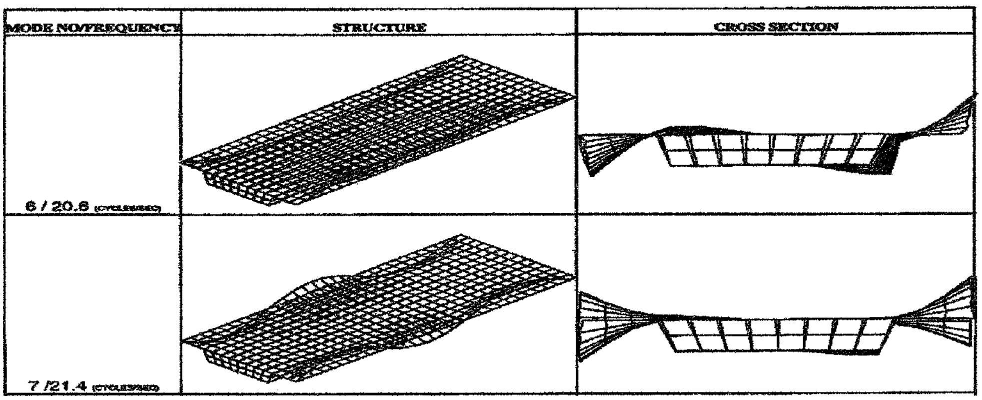
Summary of Results

Stress Analysis

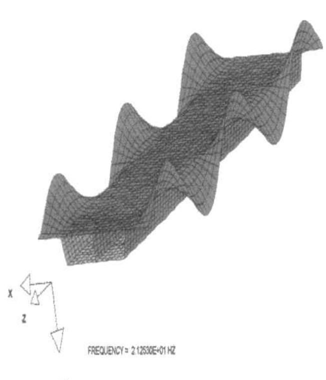
The numerical studies indicate that ratio of maximum normal stresses at flange-web junctions of the cross section (i.e., $\sigma_z/\sigma_{\mathrm{Bernoulli}}$) varies from 1.2 to 2.5 with the maximum value at points close to points of load applications and at flange-web junctures for the top flange and bottom flange plates. This nonuniform stress distribution resulting from shear-lag effects cannot be predicted using the Bernoulli beam theory, and a more refined finite element analysis is used. It is further found that, whereas the use of cross-section diaphragm has only localized effects on distributions of normal stresses σ_y (Fig. 2a), the effect on the transverse stress σ_x for the top flange and on the shear stress τ_{yz} for the middle web is more pronounced (Figs. 2b and 2c). The magnitude of σ_x is diminished to a zero value at sections of the top flange far from the loaded points, and the middle web shear stress τ_{yz} is reduced by as much as 60%. This confirms the effectiveness of the diaphragm to improve cross-section resistance to torsion for unsymmetric loads. Further analysis of results also indicates 1) for unsymmetric load cases, edge webs are less stressed compared to the middle webs; 2) nonuniformity of stress distributions is maintained in each load case, but the magnitude of stress is reduced when a diaphragm is used; 3) the locations of maximum value of transverse normal stress σ_x are identical to those of σ_z, but the maximum value of σ_x is less by nearly one order of magnitude; 4) addition of a diaphragm results in a mild increase in σ_z that does not exceed 5% in the webs, but the reduction of maximum values of σ_z in the top flange is nearly 60%.



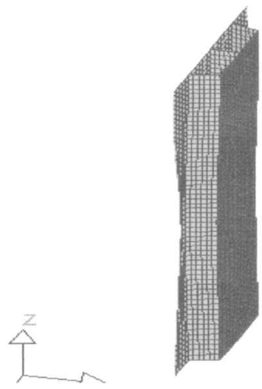
a) Effect of plate thickness ratio β on first mode shape



b) Mode shapes 6 and 7 with optimum β and a midspan diaphragm



c) Mode shape 15 without an edge beam



d) Mode shape 15 with an edge beam (axial mode)

Fig. 3 Modal characteristics of the structure with/without cross section diaphragm and/or top flange edge beam.

Modal Characteristics

The results of modal analysis reveal the degree of importance of 1) selecting an appropriate type of finite element to model the structure; 2) proper modeling of boundary conditions, and 3) inclusion of diaphragm to modify the modal characteristics of a box structure.

The first seven modal shapes are obtained using four types of plate/shell elements including a four-noded quadratic-displacement shell element with reduced integration (i.e., the S4R element that is used as a basis for numerical comparison), triangular elements S3R and STRI3, and a quadratic five-node element S4R5 (Ref. 21). These finite elements assume that transverse shear flexibility is negligible, but they differ in the way Kirchhoff's constraint (i.e., shell normal remains orthogonal to shell midsurface) is satisfied. The numerical difference (nearly 4% for lower modes and not more than 10% for higher modes) in the results obtained is attributed to the fact that STRI3 satisfies the Kirchhoff's constraint analytically, whereas the other elements satisfy the constraint only numerically. The results provided by shell element STRI3 are believed to be more reliable and accurate, and the element has been used to perform all of the numerical studies reported in this work.

The numerical results indicate the existence of an optimal proportioning of plate components of box structures in terms of ratio β (shown in Fig. 3a) that is consistent with the analytical results of β obtained from Eq. (6) for the first bending/torsion modes. With the exception of the first bending and torsion mode shapes, the values of remaining modal frequencies are closely spaced, and the modal shapes are characterized mainly by localized bending of edges of top flanges. Moreover, the cross-section diaphragm is very efficient in suppressing undesirable combination of bending, torsion, and distortion modes (Fig. 3b for mode shapes 6, and 7), whereas the use of a light edge beam is also highly efficient for eliminating localized bending in higher modes as shown in Fig. 3c for mode shape 15.

Further analysis of results indicates that 1) high stiffness of box sections makes the results insensitive to modeling details of support conditions; 2) inclusion of a cross-section diaphragm modifies mode shape 4 from a mode of combined bending and torsion to a mode of pure bending. It may therefore be concluded that there exists an optimum combination of cross-section geometry proportioning, arrangement of cross-section diaphragm(s), and/or stiffeners of plate elements to obtain favorable stress patterns and modal characteristics for a box structure.

Conclusions

The numerical results presented in this work confirm a high structural efficiency of box-girder structures that is attributed to high flexural and torsional stiffness of these structures. More desirable modal shapes of flexure and torsion are obtained if the box cross-section ratio of web to flange thickness is close to $\sqrt{3}$. The results indicate that shear-lag phenomenon plays a major role in modifying all stress distributions in cellular structures and makes the use of ordinary Bernouli beam theory invalid. The resulting nonuniformity of longitudinal and transverse stress distributions (caused by shear lag) is not negligible and should therefore be accounted for in the design of box structures to satisfy the requirements of strength and stability of all constituent elements especially at material points close to flange-web junctions. On the other hand, the analysis of free vibration characteristics of cellular structures indicates that (for a particular proportioning of cross-section plates) a judicious placement of cross-section diaphragms and/or stiffeners of top and bottom flanges is required to improve flexural and torsion rigidity of the structure. The results indicate that design of cross-section diaphragms and/or stiffeners can be optimized to obviate the development of undesirable local bending mode of vibration and/or the simultaneous development of undesirable combined mode shapes of flexure, torsion, and/or distortion vibrations. The search for optimum combinations of cross-section plates proportioning, placement of diaphragm(s), and/or plate stiffeners is now under investigation using a mathematical programming formulation.

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A. N. Palazotto
Associate Editor